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THE EXPERIMENTAL TECHNIQUE FOR OBSERVING THE TEMPERATURES DUE TO THE COUPLED THERMOELASTIC EFFECT

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and

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Abstract—A system is described which permits routine and easy measurement of the small temperature changes which occur when a material (metal) is deformed in the linearly elastic range. The temperature change produced by a 30-lb/in^2 increase in stress in aluminum can be measured. Data are reported for long thin wires and cantilever beams made of aluminum. There is generally excellent agreement with the predictions of linear thermoelasticity which is regarded as demonstrating the accuracy of the method. After 11 cycles of free vibration, the cantilever beam surface temperature leads the strain in time. There is also conduction to the neutral axis region in the case of the beam.

INTRODUCTION

THE BASIC theory of coupled linear thermoelasticity is well understood [1] although not many problems have been completely solved. There are numerous approximate, but only a few exact, solutions available in the recent literature. One example of a complete solution is given in [2] and an elegant approximation method is presented in [3]. Most investigators make the assumption that the coupling effect is negligible because a certain non-dimensional parameter is very small for most real materials. Typical arguments are given in [1] for neglecting the coupling effect. It is certainly true that in many quasi-static problems one can legitimately consider the thermal and mechanical field equations as separate problems. This is not always the case in dynamic and inelastic problems. For instance sharp discontinuities are "smoothed" out and precursors exist in dynamic linear thermoelasticity theory. In a mathematical sense the introduction of the coupling requires that additional boundary conditions be satisfied and therefore each problem must be evaluated to see whether the common uncoupling procedure is legitimate or not. However not much exists in the way of experimental data on the coupled thermoelastic effect which could guide in making the approximations.

During the past several years the senior author has been experimentally investigating the heat generated (i.e. coupled thermoplasticity [4]) during small plastic deformations of metals. The basic purpose of this study is to establish information so that the basic principle of the conservation of energy can be used in plasticity. The specimens are usually tubes which are oscillated in torsion at 1 c/s (or less). When the rate of doing plastic (mechanical) work is compared with the rate of heat generation it is found that they are not always the same and in fact either one of them can be much larger than the other. Recent modifications and improvements in technique and equipment permit the experimental facility to be used to study the linear range of the material response.

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It was thought to be desirable to experimentally study this range of the material response because there are relatively few questions about the basic theory. Therefore, the experiments described herein do not yield any new or startling developments and in fact they are rather precisely what one expects. Hence, the purpose of the paper is to describe the techniques used to measure the small temperature differences and to present typical results. The paper also demonstrates that the coupled linear thermoelastic effect can be measured. The same experimental technique is used to study inelastic deformations where the theory is not yet adequately based. These latter results are to be described elsewhere.

We have done other types of experiments than those which will be described herein. Each of the ones presented has been done many times and the data selected for photographic quality and not best accuracy.

The basic theory

The mechanism of coupled linear thermoelasticity is simply an expression of the facts that compressing a solid raises its temperature and heating causes it to expand. The energy equation for isotropic, linear thermoelasticity is [1]

$$K\Delta^2\theta = \rho C_D\,\partial\theta/\partial t + \alpha T_0(3\lambda + 2\mu)\,\partial e/\partial t \qquad (1)$$

In equation (1), θ is the temperature difference above a reference value T_0; K, C_D, and α are the thermal conductivity, specific heat and coefficient of linear thermal expansion, respectively; while λ and μ are the Lamé elastic constants. In equation (1), ρ is the undeformed density, e is the dilatation, and Δ^2 is the three-dimensional Laplacian operator. In the case of a long thin wire, equation (1) reduces to

$$K\,\partial^2\theta/\partial x^2 = \rho C_\sigma\,\partial\theta/\partial t + \alpha T_0\,\partial\sigma_{11}/\partial t \qquad (2)$$

where x is the axial coordinate and σ_{11} is the axial stress.

Experience shows that the addition of a radiation shield and the insulation of the lateral surfaces produce no significant differences from the case where the metal is exposed to air. Most of the heat loss is by axial conduction in the case of the rod in tension or in torsion with the grips serving as the heat sink. In many tests we do wrap the lateral surfaces in a Fiberglas insulation because this reduces the temperature fluctuations in the specimen due to air currents in an air-conditioned laboratory room.

By locating the thermocouple at the center of the wire in the axial direction, axial conduction can be minimized and we can compare the terms in the right-hand side of equation (2). It is obvious that if one can measure the temperature neglecting conduction (i.e., R.H.S. of (2)), that the technique can be used for measurements over longer time intervals. This permits the comparison of generation and conduction effects.

EXPERIMENTAL METHOD

The problem presented by the desire to measure the effect of the coupling of the thermal and mechanical fields is to develop a method of measuring temperature changes of the order of 0·001°F. In aluminum this temperature change is produced by a change in stress of 30 lb/in^2. The basic method which is used is to attach a thermocouple to the surface of a metallic specimen. The potential developed by the thermocouple under the initial conditions is cancelled by the addition of an equal and opposite one. The sum of these voltages is amplified by a factor of 50×10^3 and the amplifier output recorded on a suitable chart recorder or oscilloscope. A schematic of the system that is used is shown in Fig. 1. With the settings shown in this figure, the sensitivity is 0·0069°F per division of the recorder. By careful measurement of the record, the desired accuracy is obtained.

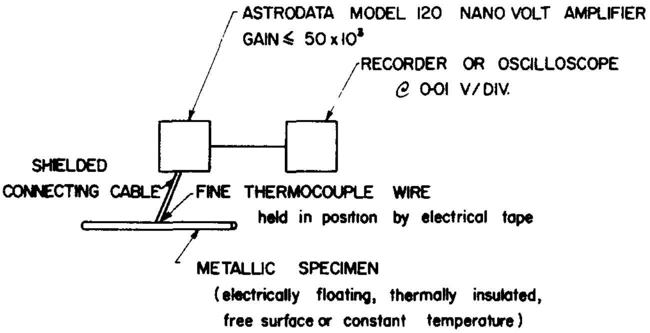


Fig. 1. A schematic of the system used to measure small temperature differences.

To convert the schematic system into a workable one the major problem is to get a good signal-to-noise ratio. In the system reported here the battery operated amplifier is the crucial item. It is also essential to have the specimen mounted in grips which are electrically floating (insulated) with respect to the rest of the system. An earlier system which did not employ a solid-state battery-operated amplifier worked but the frequency response was less than the present one. Fewer noise problems exist in the present system and once working, it gives very reproducible data. The use of shielded cables and other common ways of reducing noise (ground loops, etc.) produces a system in which there is some noise but where the desired accuracy is obtained.

To accurately follow fluctuations in temperature, the response time of the thermocouple should be small. We have fabricated thermocouples in which the response of the entire system is approximately 1 msec, but normally ones which have a 10 msec time constant are used. The major parameter in determining the response time of the thermocouple proper is the size of the junction mass. To reduce the response time, the two wires are just laid on the specimen surface and then held in position by common electrical tape. Some practice is required to perfect the technique on square bars but on round ones it is relatively easy to mount the thermocouples. For further reductions in response time, the No. 30 iron–constantan wires are reduced in size by stroking with emery paper until they break, the point of fracture then has a smaller diameter and faster response. By first locating electrical insulation on the specimen surface and running the lead wires over this, the junction mass can be made smaller still. To test the response time, such a thermocouple was made and placed on the end of a lucite rod and the wires twisted together. This combination will have a response time longer than when used on metallic specimens where the wires are not twisted together. A typical response of the lucite assembly to being plunged into water is shown in Fig. 2. Since the time scale of this figure is 10 msec per division, this particular thermocouple and recorder system had a response time of less than 10 msec. Part of this time is attributable to the speed at which we could plunge the lucite assembly into the water.

As mentioned earlier, this system will respond to the changes in the specimen temperature which occur in an air-conditioned room. This "wandering" can be greatly reduced by insulating the specimen and test apparatus in Fiberglas insulation. It is also clear that one must allow the entire apparatus to equilibrate, after installation or other handling, before measurements are taken. Furthermore, personnel working on the experiment must stay, more or less, in the same position during a test. It is noted that there is one very positive experimental advantage to using two loose wires as the thermocouple. If either wire is not in good contact with the specimen, a very large amount of noise is detected. Therefore, erroneous data can not be taken when the mounting is improper.

EXPERIMENTAL RESULTS

Thin wire

To illustrate the technique a 6 ft long, $\frac{1}{8}$ in. dia., aluminum wire was prestressed in tension. Best results were obtained by applying the load gradually, allowing the system to fully equilibrate, and then cutting the wire which is holding the load. For this experiment, the strain and temperature were recorded as a function of time on an oscilloscope. Typical results are shown in Fig. 3 where the top trace is the temperature (getting hotter) and the bottom one is the strain (compressing) at approximately the same axial location.

In the example shown in Fig. 3, the change in stress is 2860 lb/in^2 and the "flat" in the temperature corresponds to a change of 0·093°F. Therefore the change in energy given by $\alpha T_0 \Delta\sigma$ is 19·4 in-lb/in^3 while the change in thermal energy $\rho C_\sigma \Delta\theta$ is equal to 19·2 in-lb/in^3. Not all data are in such perfect agreement, but most of the discrepancies are equal to the error in reading the data. The specimen then slowly cools.

When the load is gradually applied in tension, somewhat smaller temperature differences are observed than those shown in Fig. 3. Presumably this is due to heat conduction effects and does not imply a difference between tension and compression. When the tensile load is "dropped" onto the specimen, the results are similar to those shown in Fig. 3. Experimental data for the case where the end of the specimen is attached to the load by means of a thin (weak) string are shown in Fig. 4. When the load is dropped and while the string is unbroken, a tensile pulse is applied to the specimen. When the string breaks, an unloading wave is introduced. The polarity of the temperature signal was inverted on the oscilloscope for Fig. 4, so that the pulse shapes could be compared. For the major part of the time shown in Fig. 4, the specimen is in tension and the temperature is decreased from ambient. Because of the difference in scale factors, the temperature is as "large" as the strain. There is some delay between the strain and temperature rises, with the strain leading in time. Presumably this is due to the thermocouple and not to the material response. However, it is a bit curious that there should be a difference in the delay between the upper and lower frames of Fig. 4. The setups were thought to be exactly the same in the two tests.

Cantilever beam

The very close agreement between the temperatures generated by deformation processes in cases like the one shown in Fig. 3 caused us to wonder if this could be expected in a situation where the strain was nonuniform. An elementary case where the strain is nonuniform is the beam in bending, and cantilever boundary conditions were used in the

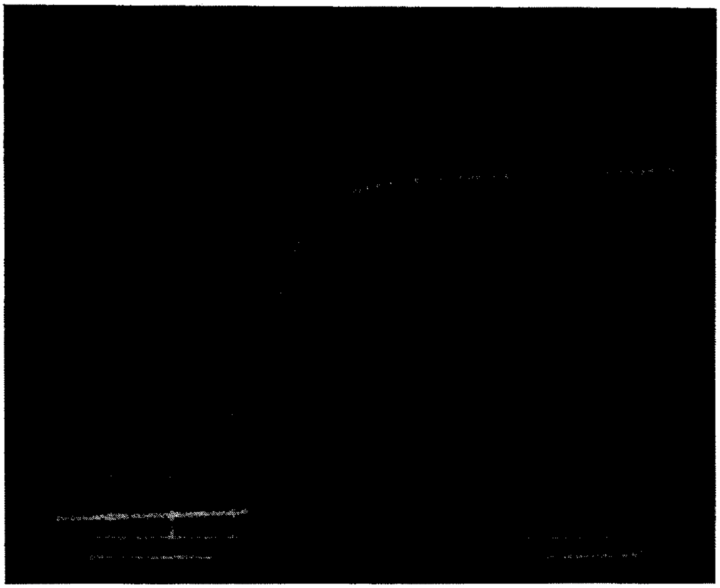


FIG. 2. The response of a thermocouple mounted on a lucite rod upon being plunged into a water bath. The time scale is 10 msec per division.

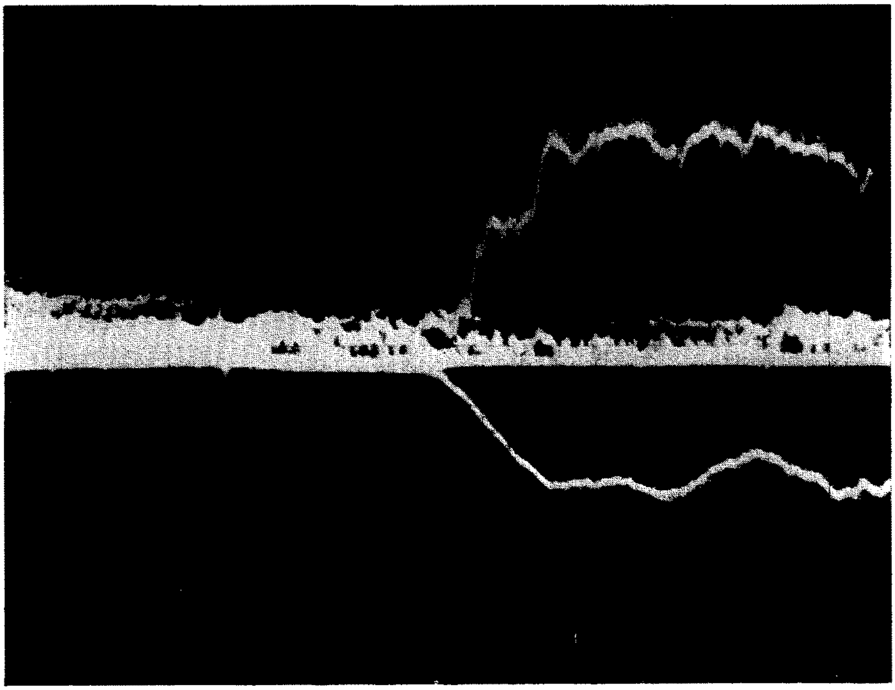


FIG. 3. Experimental data showing the time history of the change in temperature (top trace) and strain (bottom trace) of a long wire of aluminum when a stress of 2860 lb/in^2 is " cut off ". The change in temperature is 0·093°F. The temperature scale is 0·042°F per division.

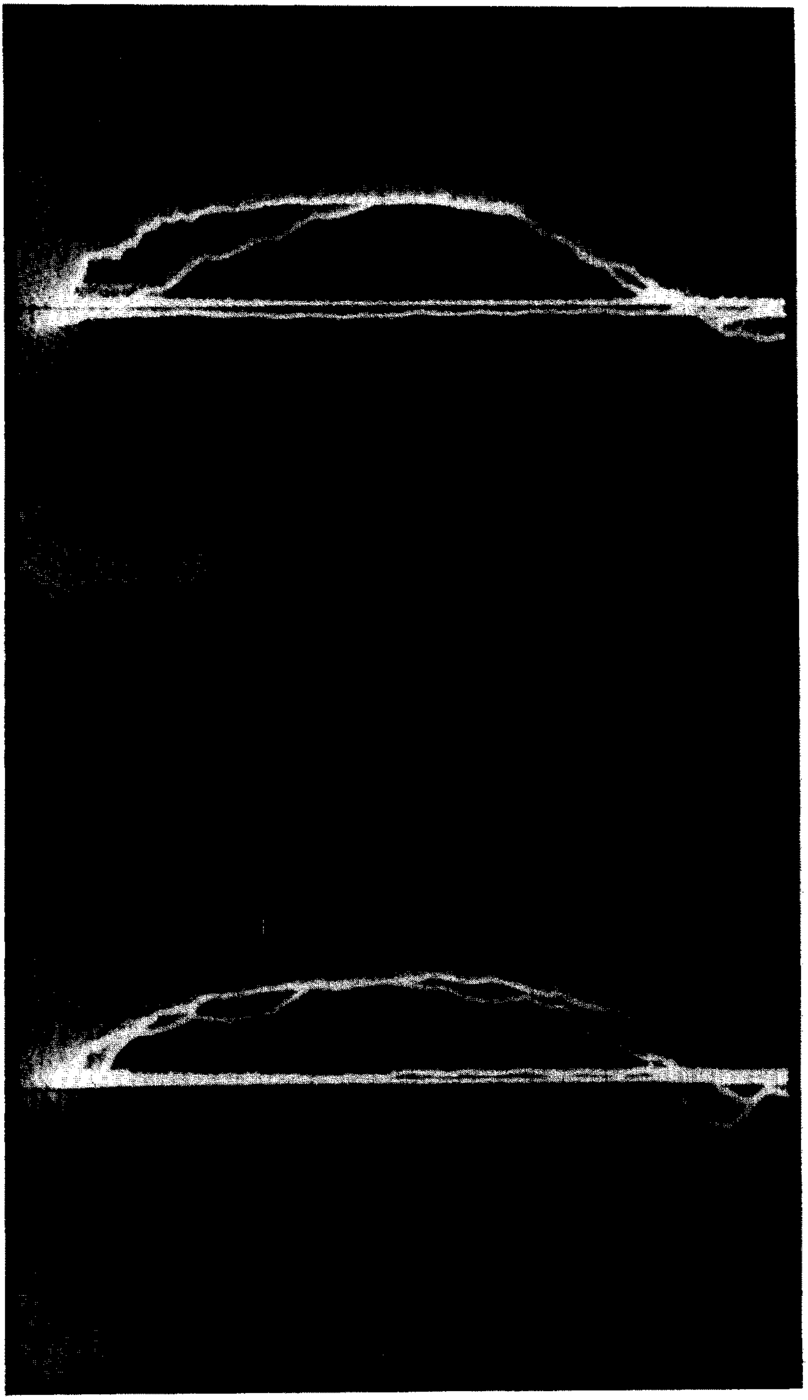
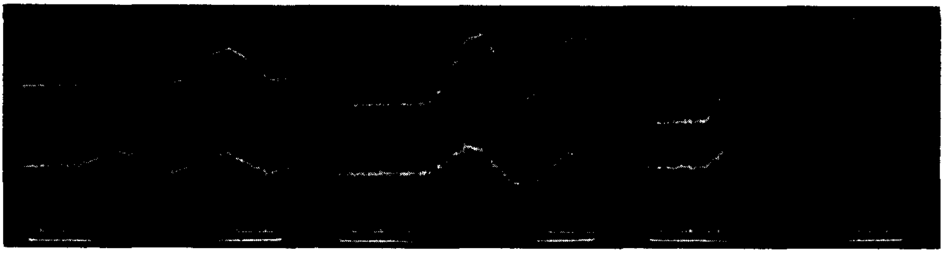


Fig. 4. Experimental data on the change in temperature (trace which is lagging) and strain (leading trace) as a function of time. In this test the load was connected to a string which breaks and unloads the specimen. The time scale is 1 msec per division. The upper and lower pictures were meant to be identical tests. The purpose of the figure is to show that the temperature follows the strain rather well. Note: The temperature signal polarity was inverted at the oscilloscope as compared to Fig. 3.



(a) (b) (c)

FIG. 5. Experimental strain and temperature histories for a cantilever beam when three different loads are "cut off." The left frame (a) is for a change in load corresponding to a static change in stress of 2170 lb/in^2; the centre frame (b) to 4340 lb/in^2; the right-hand frame (c) to 6500 lb/in^2. In each case the upper trace is the strain history and the bottom is the temperature history at points approximately 1 in. apart. The temperature increases linearly with the strain amplitude. The time scale is 10 msec per division and the temperature is 0·086°F per division for all frames.

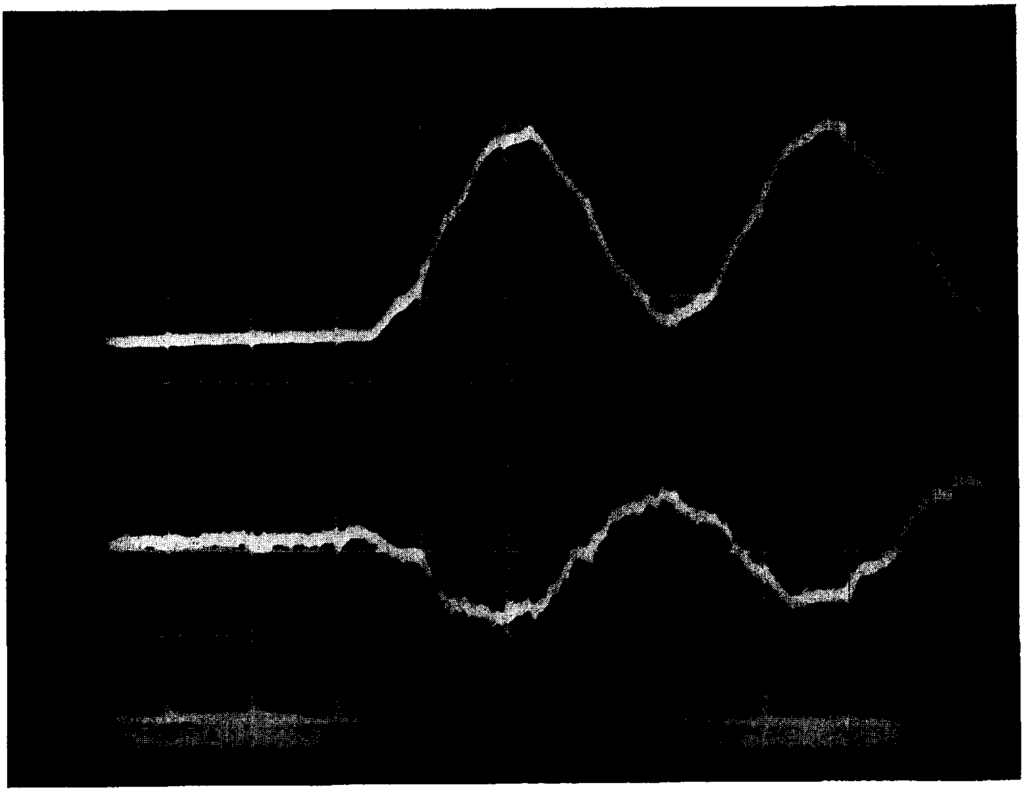


FIG. 6. Experimental strain–time history of the temperature on the bottom surface of the cantilever beam. In all other respects this test is the same as in Fig. 5.

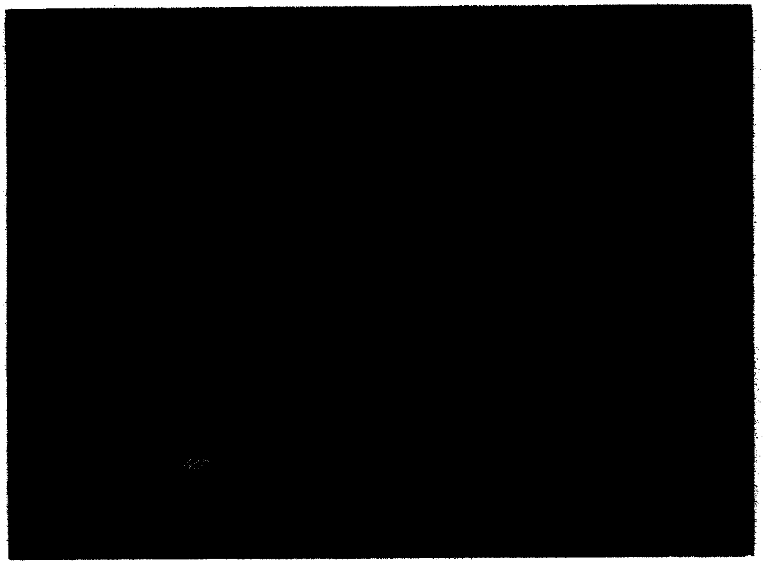


FIG. 8. A typical history of an annealed aluminum tube after several oscillations in torsion. These oscillations are large enough that the material is plastic. The time scale is 1 sec per division. Notice that the temperature is constant during "unloading". Therefore, adiabatic conditions exist during the time interval of this test. The two upper traces are strain at two points along the tube, while the bottom one is the temperature. The next to the bottom (initially) trace is the load which is convertible into average stress. The time scale is 1 sec per division and the temperature sensitivity is 0·086°F per division.

experiment. The method of preloading the specimen and cutting off the load is used again.

The specimen is a $\frac{1}{4}$ in. × $\frac{1}{4}$ in. square aluminum bar, 35 in. long mounted as a cantilever beam in electrically floating supports. The thermocouple is located 5 in. from the fixed end and a strain gauge mounted 6 in. from this end. In the first set of experiments both are located on the top surface. Experimental results for three values of the load are shown in Fig. 5. The thermocouple was then moved to the bottom surface of the specimen at the same axial location as before, while the strain gauge remained unchanged from that shown in Fig. 5. The result for the same load as used in Fig. 5(b) is shown in Fig. 6. The temperature changes in Figs. 5(b) and 6 are very nearly the same. Figures 5 and 6 are presented in order to demonstrate the linearity of the phenomena. This is clearly seen to be the case.

The first peak in the strain and temperature occur at the same time in either Figs. 5 or 6, thereby indicating negligible phase difference in the initial cycle. However, later on (beyond the times represented by these photographs) there is a measureable difference in phase and the temperature *leads* the strain. This is not due to the slight time difference for the wave to propagate the one inch between the thermocouple and the gauge. It is also unlikely that it is due to the temperature sensitivity of the strain gauge because the greatest temperature difference is in the first cycle. Typical results are shown in Fig. 7, which is the long time response of the experiment presented as Fig. 5(c). The temperature is seen to lead the strain by about 0·01 sec. This test takes 45 sec to equilibrate.

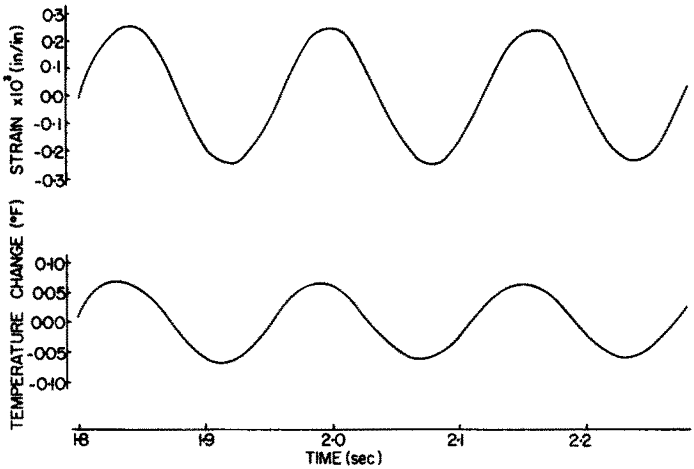


FIG. 7. The strain and temperature as a function of time for the test shown as Fig. 5(c), but after 11 cycles. The maxima in the temperature occur slightly earlier than those in the strain.

The peak in strain in the first cycle of Fig. 5(c) corresponds to a compressive change in stress of 6500 psi at the thermocouple location. This value was determined by using a static calibration, strength of materials and the dynamic strain measurement. The temperature increase is 0·129°F during this first quarter cycle. Thus the change in $\alpha T_0 \Delta\sigma$ is 44·0 in-lb/in^3 while that in $\rho C_\sigma \Delta\theta$ equals 27·4 in-lb/in^3.

Thus there appears to be conduction to the neutral axis region and therefore one cannot completely neglect this phenomenon when the strain is not uniform.

Thermoplastic torsion

In order to illustrate the use of the experimental procedure described above when the results are less obvious, Fig. 8 is presented. In this experiment the specimen is oscillated in torsion. The load (stress), strain at two points, and the temperature in a round tube are measured and recorded as shown in Fig. 8. From this data one can evaluate the rate of doing plastic work and the rate of generating heat. The results show that over a complete cycle, the work exceeds the heat generation. Energy is therefore stored by the internal structural changes but this is not necessarily the case at a given instant of time. The rate of heat generation can greatly exceed the rate of doing plastic work and vice versa.

DISCUSSION

A technique has been described which permits the measurement of the small temperature changes which occur in the linear range of the material response. The results show that there is an accurate agreement with the predictions of linear thermoelasticity. This is not surprising but is useful, because it demonstrates the accuracy of an experimental method which should be applicable to situations in which the theory is not so well formulated (e.g. inelastic response).

The particular technique reported herein is limited to making one temperature measurement per specimen because the amplifier requires electrical isolation of each specimen. In order to extend this technique it will be necessary to develop a good thermal connection which retains the electrical insulation. Many different metals have been used with this technique. For non-conductors, the thermocouple wires must be joined and therefore the response time is somewhat reduced.

The temperature changes which are developed are not a large number of degrees so that they will not cause failure due to temperature *per se*! However, this does not mean that coupling is completely negligible. Its role in linear thermoelasticity is like damping in a linear oscillator. In some problems it is an extremely accurate approximation to neglect it, but in other problems it will be necessary to retain the coupling phenomena.

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Résumé—Un système est décrit qui permet de mesurer routinièrement et facilement les changements des petites températures qui surviennent lorsqu'une matière (métal) est déformée sur l'étendue linéaire élastique. Le changement de température produit par une augmentation de 30 (trente) psi (livres par inch carré) de tension d'aluminium peut être mesuré. Des données sont établies pour des fils métalliques longs et fins et des poutres en console faits en aluminium. Il y a en général une excellente conformité avec les prédictions de thermoélasticité linéaire qui est considérée comme démontrant l'exactitude de la méthode. Après onze (11) cycles de vibrations libres, la température de surface des poutres en console conduit la tension avec le temps. Il y a également conduction vers la région de l'axe neutre dans le cas de poutre.

Zusammenfassung—Es wird ein System beschrieben welches die routine mässige und einfache Messung von geringen Temperaturveränderungen erlaubt, die vorkommen, wenn ein Material (Metall) in linearen elastischen Gebiet deformiert ist. Die Temperaturveränderung welche bei einer Dreissig, (30) Pfund/Quadretzoll Erhöhung der Beanspruchung in Aluminium erzeugt wird, kann gemessen werden. Angaben für lange dünne Drähte und Kragträgern aus Aluminium werden gegeben. Gewöhnlich ist eine ausgezeichnete Übereinstimmung mit den Resultaten der linearen Thermoelastizität vorhanden, welche als ein Beweis für die Genauigkeit der Methode angesehen wird. Nach Elf (11) freien Schwingungsdrehungen, führt die Kragträgeroberflächentemperatur in Zeit zu einer Beanspruchung. Fluss zum Gebiet der Neutralachse ist im Falle des Trägers ebenfalls vorhanden.

Абстракт—Описывается система, которая позволяет повседневное и лёгкое измерение малых изменений температур, которые получаются, если материал (металл) деформируется в линейно упругой зоне. Возможно измерение изменения температуры, которое происходит при увеличении напряжения в алюминии на тридцать (30) psi = фунтов на квадратный дюйм. Доложены данные для длинных тонких проволок и для консольных балок, сделанных из алюминия. Существует общая прекрасная согласованность с предсказаниями линейной термозластичности, что рассматривается, как указание на точность метода. После одиннадцати (II) циклов свободной вибрации температура поверхности консольной балки проводит напряжение во времени. В случае балки существует также проводимость к району нейтральной оси.